DYNAMIC CHARACTERICTICS OF MEASUREMENT SYSTEMS

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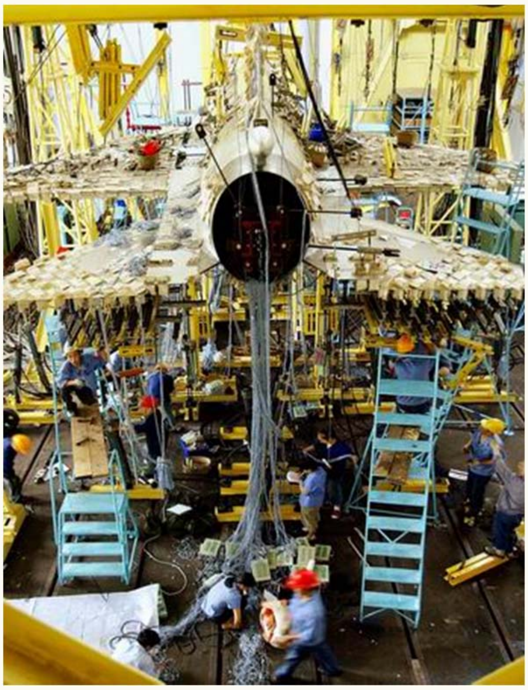
NTU50235100

Airframe static test



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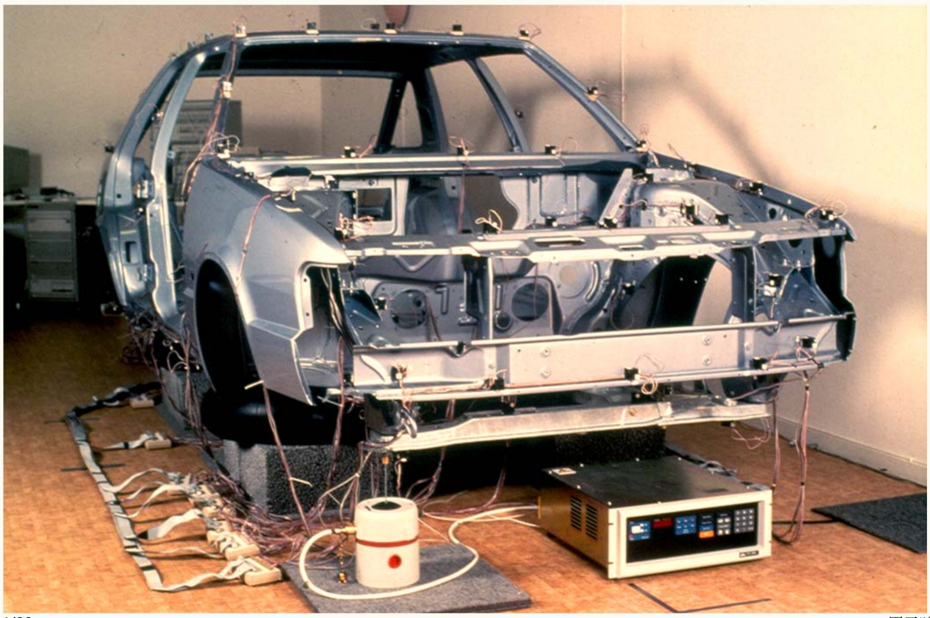
Passenger car modal test



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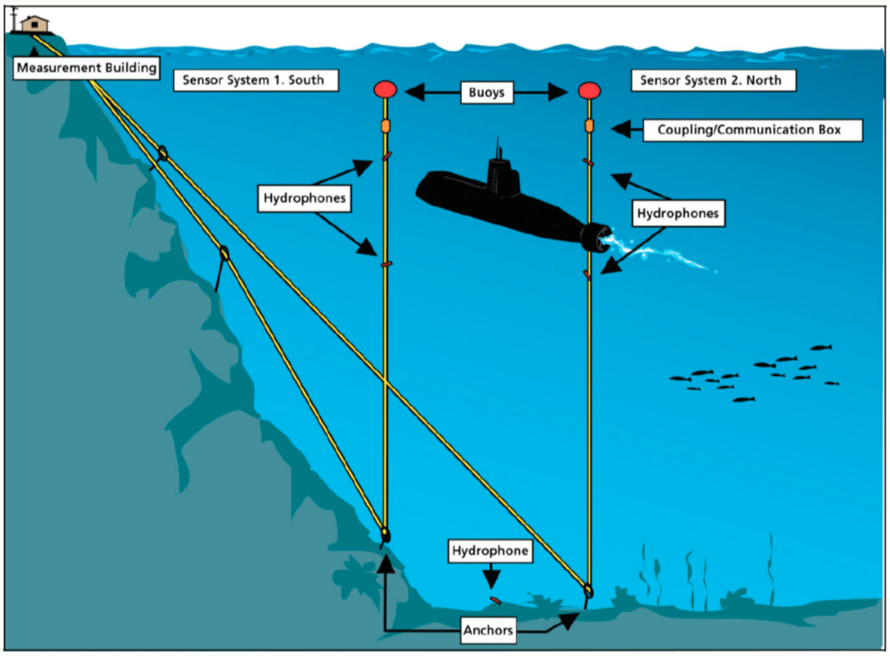
Submarine detection



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Passenger car noise measurement

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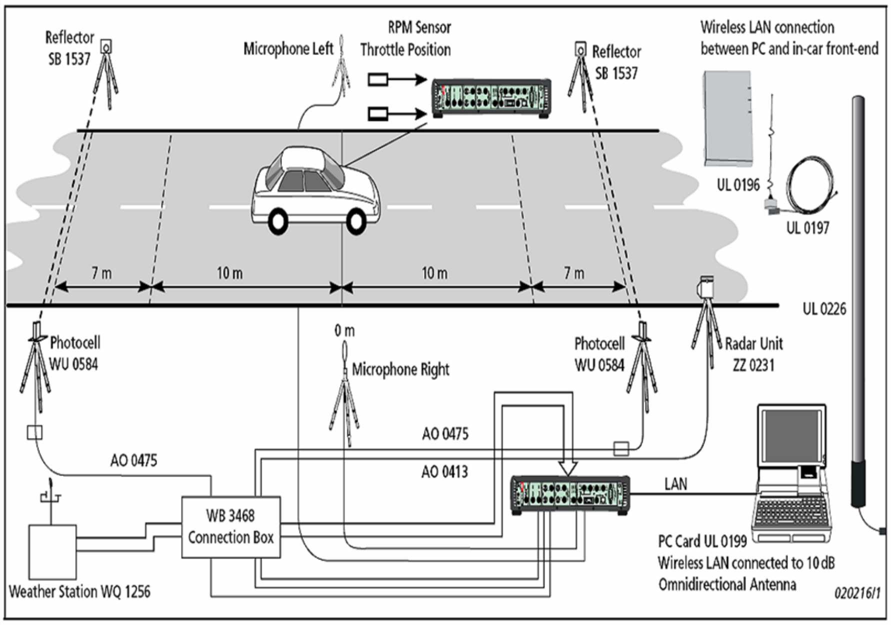

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Generalized Mathematical Model

$q_i(t)$ → Measurement system → $q_o(t)$

Input Output

$$a_n \frac{d^n q_o}{dt^n} + a_{n-1}\frac{d^{n-1} q_o}{dt^{n-1}} + \cdots + a_1 \frac{dq_o}{dt} + a_0 q_o = b_m \frac{d^m q_i}{dt^m} + b_{m-1}\frac{d^{m-1} q_i}{dt^{m-1}} + \cdots + b_1 \frac{dq_i}{dt} + b_0 q_i$$

q_o : output quantity

q_i : input quantity

t : time

a's, b's : physical parameters

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- Operational transfer function

Differential operator: $D = \dfrac{d}{dt}$

$$\left(a_n D^n + a_{n-1} D^{n-1} + \cdots + a_1 D + a_0\right) q_o = \left(b_m D^m + b_{m-1} D^{m-1} + \cdots + b_1 D + b_0\right) q_i$$

$$\frac{q_o}{q_i}(D) = \frac{b_m D^m + b_{m-1} D^{m-1} + \cdots + b_1 D + b_0}{a_n D^n + a_{n-1} D^{n-1} + \cdots + a_1 D + a_0}$$

- Laplace transfer function

$$\frac{\overline{q}_o(s)}{\overline{q}_i(s)} = G(s) = \frac{b_m s^m + b_{m-1} s^{m-1} + \cdots + b_1 s + b_0}{a_n s^n + a_{n-1} s^{n-1} + \cdots + a_1 s + a_0}$$

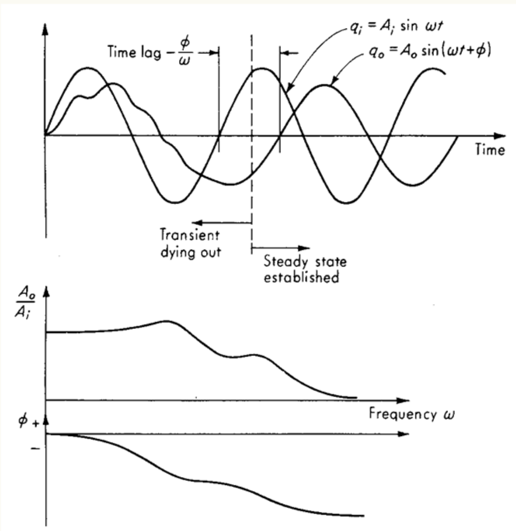
$$s = \sigma + i\omega$$

- Sinusoidal transfer function (Frequency response function)

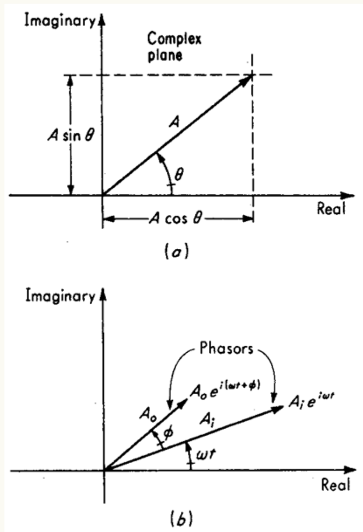
$$G(\omega) = \frac{\tilde{q}_o(\omega)}{\tilde{q}_i(\omega)} = \frac{b_m (i\omega)^m + b_{m-1} (i\omega)^{m-1} + \cdots + b_1 (i\omega) + b_0}{a_n (i\omega)^n + a_{n-1} (i\omega)^{n-1} + \cdots + a_1 (i\omega) + a_0}$$

Input: $q_i(t) = A_i \sin \omega t$ Output: $q_o(t) = A_o \sin(\omega t + \phi)$

A_o / A_i: amplitude ratio ϕ: phase angle $G(\omega) = \left(A_o / A_i\right) e^{i\phi}$



Frequency response function



Phasor representation

Requirements for accurate measurement

- Single frequency input

$$\text{Input}: \; q_i = A_i \sin \omega_0 t \quad , \quad \text{Output}: q_o = A_o \sin(\omega_0 t + \phi_o)$$

$$\text{Gain}: \; \frac{A_o}{A_i} = |G(\omega_0)| \quad , \quad \text{Phase}: \; \phi_0 = \angle G(\omega_0)$$

$$q_o(t) = A_i |G(\omega_0)| \sin \omega_0 (t - \tau_0)$$

$$\tau_0 = -\frac{\angle G(\omega_0)}{\omega_0} \; : \; \text{Dealy time}$$

For measured output $q_o = A_o \sin \omega_0 t$

The corresponding input $q_i(t) = \dfrac{A_o}{|G(\omega_0)|} \sin \omega_0 (t + \tau_0)$

- Input with two frequencies

$$q_i = A_{i1} \sin \omega_1 t + A_{i2} \sin(\omega_2 t + \theta_2)$$

$$q_o = A_{o1} \sin(\omega_1 t + \phi_1) + A_{o2} \sin(\omega_2 t + \theta_2 + \phi_2)$$

$$\frac{A_{o1}}{A_{i1}} = |G(\omega_1)| \quad , \quad \phi_1 = \angle G(\omega_1) \qquad \frac{A_{o2}}{A_{i2}} = |G(\omega_2)| \quad , \quad \phi_2 = \angle G(\omega_2)$$

$$q_o = A_{i1} |G(\omega_1)| \sin\left(\omega_1 t + \angle G(\omega_1)\right) + A_{i2} |G(\omega_2)| \sin\left(\omega_2 t + \angle G(\omega_2) + \theta_2\right)$$

A good measurement system should provide $q_o(t) = C q_i (t - \tau)$

or $q_o = K\left[A_{i1} \sin \omega_1 (t - \tau) + A_{i2} \sin\left(\omega_2 (t - \tau) + \theta_2\right)\right]$

It can be satisfied if

$|G(\omega_1)| = |G(\omega_2)| = K$ and $\dfrac{\angle G(\omega_1)}{\omega_1} = \dfrac{\angle G(\omega_2)}{\omega_2} = -\tau$

- Requirements for accurate measurement

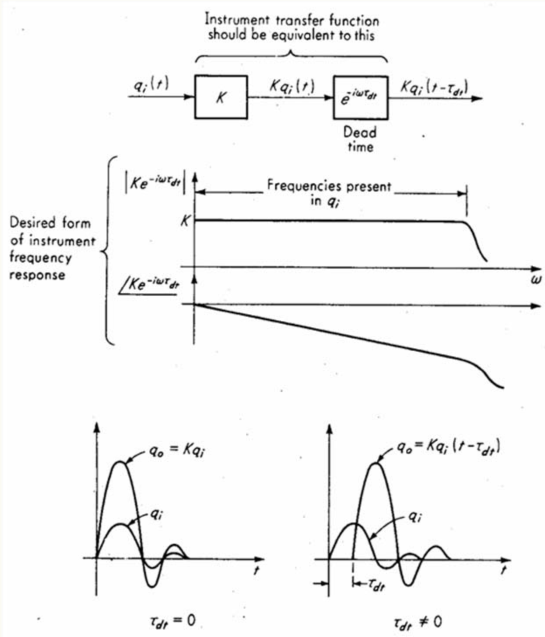
$$|G(\omega)| = \text{constant}$$

Constant gain

and

$$\angle G(\omega) = -\tau\omega$$

Linear phase



Requirements for dynamic measurement

- Zero-order instrument

$$a_0 q_o = b_0 q_i$$

$$q_o = \frac{b_0}{a_0} q_i = K q_i$$

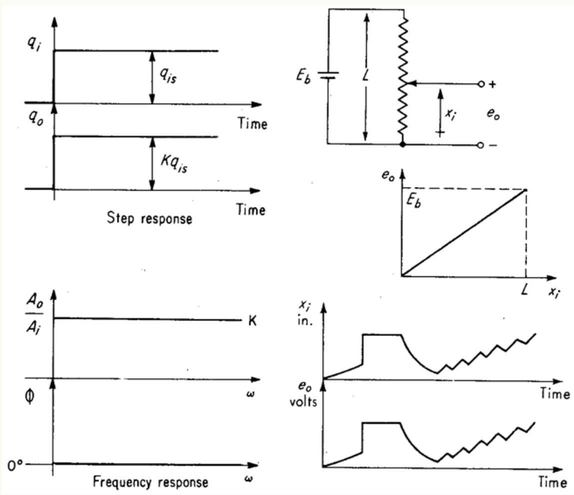
K: static sensitivity

Examples:

(i) Strain gage

$$\frac{dR}{R} = G\varepsilon$$

G: gage factor



(ii) Potentiometer $\quad e_o = \frac{x_i}{L} E_b = K x_i$

Perfect instrument for dynamic measurements

- First-order instrument

$$a_1 \frac{dq_o}{dt} + a_0 q_o = b_0 q_i \quad \text{or} \quad \tau \frac{dq_o}{dt} + q_o = K q_i$$

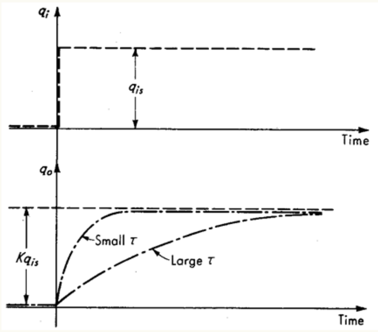
$$\tau = \frac{a_1}{a_0} \text{ : time constant,} \qquad K = \frac{b_0}{a_0} \text{ : static sensitivity}$$

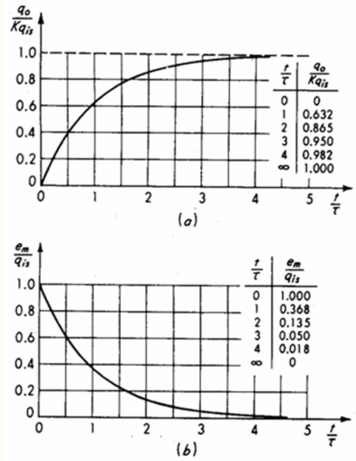
Step response

$$q_o = K q_{is} (1 - e^{-t/\tau})$$

measurement error:

$$e_m = q_i - \frac{q_o}{K}$$





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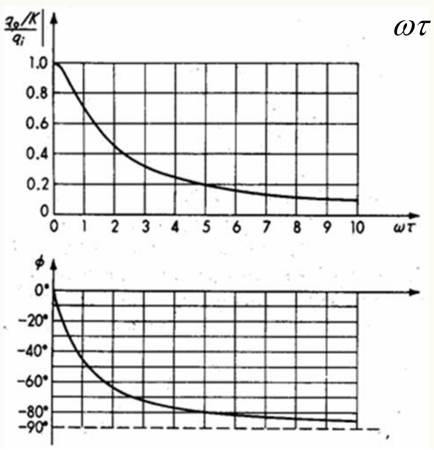
Frequency response $G(\omega) = \dfrac{K}{i\omega\tau + 1}$

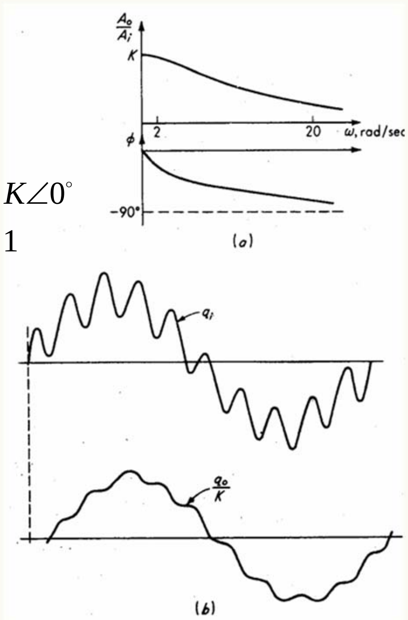
$$\frac{A_o}{A_i} = |G(\omega)| = \frac{K}{\left(\omega^2\tau^2 + 1\right)^{1/2}}$$

$$\phi = \angle G(\omega) = \tan^{-1}(-\omega\tau)$$

Ideal frequency response function $G(\omega) = K\angle 0^\circ$

$$\omega\tau << 1$$





Inadequate frequency response

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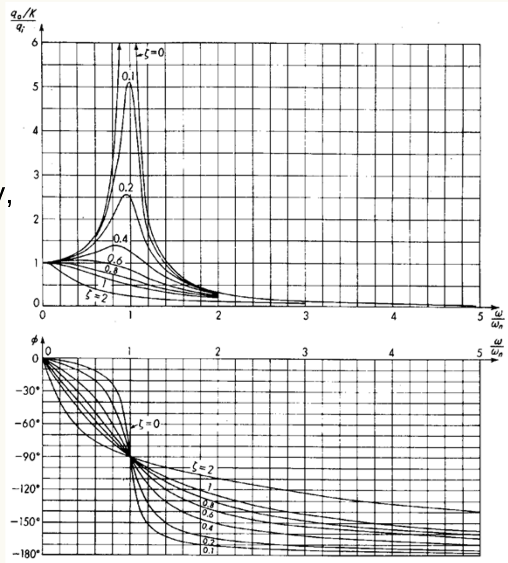
- Second-order instrument

$$a_2 \frac{d^2 q_o}{dt^2} + a_1 \frac{dq_o}{dt} + a_0 q_o = b_0 q_i$$

$K = b_0 / a_0$: static sensitivity,

$\omega_n = (a_0/a_2)^{1/2}$: natural frequency,

$\zeta = \dfrac{a_1}{2(a_0 a_2)^{1/2}}$: damping ratio



$$G(\omega) = \frac{K}{1 + i2\zeta(\omega/\omega_n) - (\omega/\omega_n)^2}$$

$$\phi = \tan^{-1}\{2\zeta / [(\omega/\omega_n) - (\omega_n/\omega)]\}$$

$$|G(\omega)| = \frac{K}{\left\{\left[1-(\omega/\omega_n)^2\right]^2 + 4\zeta^2(\omega/\omega_n)^2\right\}^{1/2}}$$